Electromagnetic Scattering of the Carbon Nanotubes Excited by Electric Line Source

Y. Wang[1,2], Q. Wu[1,3], Y. M. Wu[4], L. W. Li[4], Y.Wang[2], J. Y. Dong[2], and P. Wang[2] , X. J. He[2]
[1]Department of Science and Technology, Harbin Institute of Technology, Harbin, 150001, China
[2]Department of Science and Technology, Harbin University of Science and Technology, Harbin, 150080, China
[3]State Key Laboratory of Millimeter Wave, Nanjing, 210096, China
[4]Department of Electrical and Computer Engineering, National University of Singapore, Singapore 119260, Singapore
E-mail: wsbte@126.com

Abstract — In this paper an analytical solution is presented for the electromagnetic scattering from infinite-length metallic carbon nanotube and carbon nanotube bundle. The scattering field and scattering cross section are predicted using a modal technique based on a Bessel and Hankel function for the electric line source, and a quantum conductance function for the carbon nanotube. The particular case of an isolated armchair (10, 10) carbon nanotube, the scattered field predicted from this technique is in excellent agreement with the measured result. Furthermore, the analysis indicates that the scattering pattern of an isolated carbon nanotube differs from that of the carbon nanotube bundle formed by the same index (m, n) carbon nannotube.

I. INTRODUCTION

SWNTs (SWNTs) comprise a family of more than several hundred structures characterized by different diameters and helicity which determine their electronic structure and optical properties [1]. They can be either metallic or semiconducting dependant sensitively on atomic structure. This richness and diversity make carbon nanotubes so promising for various applications. In this paper we consider electromagnetic scattering from infinite-length metallic SWNTs, and carbon nanotube bundle. The homogeneous carbon nantobe bundle is comprised of the same armchair nanotubes. The model utilized here combines an effective boundary condition to describe the wave propagation around carbon nanotubes, together with the quantum conductance function for the carbon nanotubes[2].

During the past years, different theoretical model have been used to describe electromagnetic wave scattering by carbon nanotubes and other scattering configuration [3]-[7].

The Recent studies on the Rayleigh scattering spectra of SWNTs provide spectroscopic information about the system through the scattering resonant enhancement when the photon energy matches that of an electronic transition. These data permit us to test key theoretical constructs concerning the scattering properties of carbon nanotubes [8].

This paper presents an exact analytical solution of the scattering of cylindrical waves from infinitely long metallic SWNTs and bundle excited by an electric line source. The analysis shows that the scattering intensity peak shifts and narrows the line shape in bundle, compared to a SWNT case. Nanotube bundle radius plays critical roles in governing the scattering characteristics. It is also shows that relativety large amplitude scattered field can be obtained from carbon nanotube bundle.

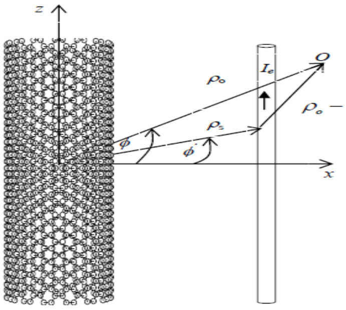

Fig. 1. A carbon nanotube illuminated by an electric line source located in free space.

II. ANALYTIC DEVELOPMENT

The SWNTs illuminated by an electric line source with a constant electric current I_e placed parallel to the nanotube, is depicted Fig. 1. The carbon nanotube is located in the cylindrical coordinate system with the associated Cartesian coordinate system as shown in the figure. It is of infinite extent in the z-direction, has a radius of a, and is centered at the origin of the coordinate systems. The ambient medium is free space. The position vector of a given observation point is ρ_o , while the position vector of the electric line source is ρ_s .

The incident field generated by the source is given by

$$E_z^i=-\frac{\omega\mu_0 I_e}{4}\times\sum_{m=-\infty}^{\infty}\begin{pmatrix}J_m(k_0\rho_o)H_m^{(2)}(k_0\rho_s)e^{jm(\varphi-\varphi')}\ \text{for}\ (\rho_o<\rho_s)\\ J_m(k_0\rho_s)H_m^{(2)}(k_0\rho_o)e^{jm(\varphi-\varphi')}\ \text{for}\ (\rho_o>\rho_s)\end{pmatrix} \qquad (1)$$

$$E_\phi^i=E_\rho^i=0\,. \qquad (2)$$

Where the function $J_m(x)$ is the Bessel function of the first kind, while $H_m^{(2)}(x)$ is the m-order Hankel function of the second kind. The scattering electric field E_z^s and transmitting field E_z^t due to the existence of the SWNT can then be written as

$$E_z^s=-\frac{\omega\mu_0 I_e}{4}\sum_{m=-\infty}^{\infty}S_m H_m^{(2)}(k_0\rho_o)e^{jm(\phi-\phi')} \qquad (3)$$

$$E_z^t=-\frac{\omega\mu_0 I_e}{4}\sum_{m=-\infty}^{\infty}T_m J_m(k_0\rho_o)e^{jm(\phi-\phi')}\,. \qquad (4)$$

where S_m and T_m are the unknown coefficient. The effective boundary conditions are then applied at the carbon nanotube surface in order to determine S_m and T_m, and final result is

$$S_m = \frac{-\eta\sigma_{cn}(\omega)[J_m(k_0a)]^2 H_m^{(2)}(k_0\rho_s)}{2/(\pi k_0 a) + \eta\sigma_{cn}(\omega) J_m(k_0a) H_m^{(2)}(k_0a)} \tag{5}$$

$$T_m = \frac{2/(\pi k_0 a) H_m^{(2)}(k_0\rho_s)}{2/(\pi k_0 a) + \eta\sigma_{cn}(\omega) J_m(k_0a) H_m^{(2)}(k_0a)} \tag{6}$$

where the quantum conductivity $\sigma_{cn}(\omega)$ of carbon nanotube is given by

$$\begin{aligned}\sigma_{cn}(\omega) = \frac{je^2\omega}{\pi^2\hbar a}\Bigg\{ & \frac{1}{\omega(\omega - j\nu)} \sum_{s=1}^{m} \int \frac{\partial F_c}{\partial p_z}\frac{\partial \varepsilon_c}{\partial p_z} \\ & -2\sum_{s=1}^{m}\int dp_z \varepsilon_c \,|R_{\upsilon c}|^2 \frac{F_c - F_\upsilon}{\hbar^2\omega(\omega - j\nu) - 4\varepsilon_c^2}\Bigg\}.\end{aligned} \tag{7}$$

The details of the expressions for these quantities have been omitted and can be obtained in [2]. From (1), (3) and (5), the scattering width is given by

$$\begin{aligned}\sigma_{2-D} &= \lim_{\rho_o\to\infty}\left(2\pi\rho_o \frac{\left| -\frac{\omega\mu_0 I_e}{4}\sqrt{\frac{2j}{\pi k_0 \rho_o}} e^{jk_0\rho_o} \sum_{m=-\infty}^{\infty} j^m S_m e^{jm(\phi-\phi')}\right|^2}{\left| -\frac{\omega\mu_0 I_e}{4} H_0^{(2)}(k_0|\rho_o - \rho_s|)\right|^2}\right) \\ &= \frac{4}{k_0}\frac{\left|\sum_{m=-\infty}^{\infty} j^m S_m e^{jm(\phi-\phi')}\right|^2}{\left|H_0^{(2)}(k_0|\rho_o - \rho_s|)\right|^2}\end{aligned} \tag{8}$$

which is dependent on ϕ and even for small radius nanotube.

III. RESULTS

To test the validity of our solution, the metallic armchair (10, 10) carbon nanotube under considered is compared with the exact Rayleigh spectroscopy experiments. Fig. 2 shows that the scattering cross section predicted from (8) is in excellent agreement with the measurement result in the range 1.7-2.6eV. The Rayleigh spectrum of (10, 10) carbon nanotube exhibits a well-defined resonant characteristic and line shape, which are perfectly represented by our analysis. The validity of this prediction is important not only for fundamental understanding of the scattering properties of SWNTs but also because of its critical role in nano antenna in lower infrared and THz bands.

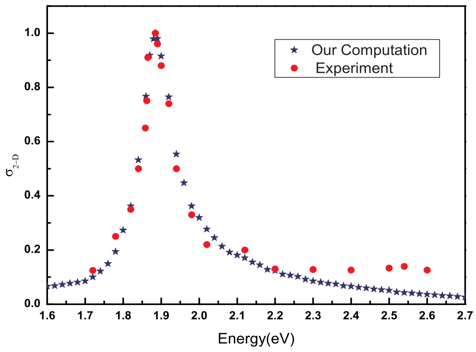

Fig. 2. Comparison scattering width between measurement and theory.

Figures 3 and 4 show that the normalized scattering width of armchair (40, 40) SWNT and bundle for $\phi' = 0^\circ$ and $\phi \in [90^\circ, 94^\circ]$, respectively. In the both case, the scattering width is dependant on ϕ, and when $\phi = 92^\circ$, σ_{2-D} approaches its maximum. It can be observed that the scattering intensity peak shifts and narrows the line shape in bundle, compared to a SWNTs case.

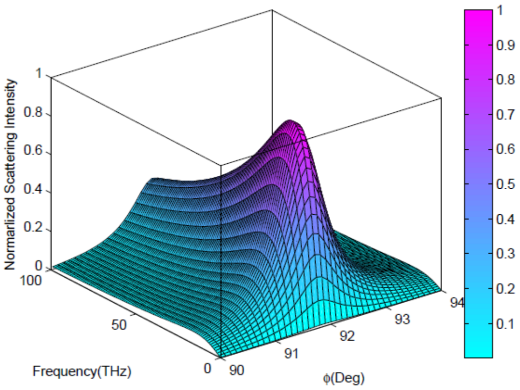

Fig. 3. Scattering feature of armchair carbon nanotube with a =2.712nm.

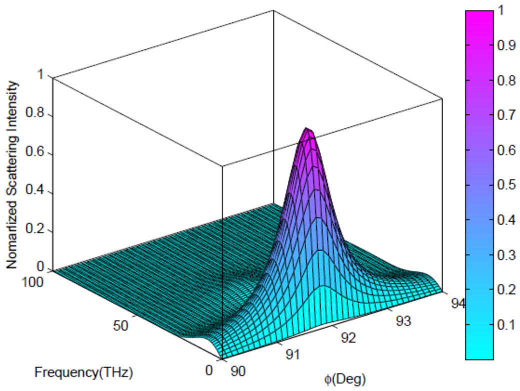

Fig. 4. Scattering feature of nanotube bundle with a =2.7 um.

IV. REFERENCES

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